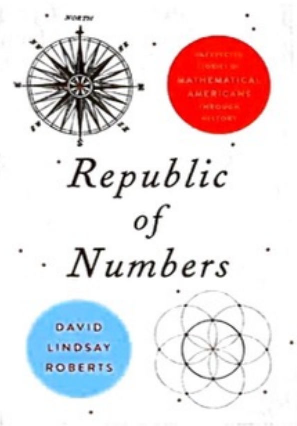
## Republic of Numbers

### David Lindsay Roberts

JOHNS HOPKINS UNIVERSITY PRESS 2019, 252 PAGES
PRICE (HARDBACK) £22.00 ISBN 978-1-4214-3308-0

**Roberts gives an account** of mathematical expansion in the United States over a 200 year period. The story is told through the biographies of 23 individual practitioners over 20 chapters. However, he does not claim to provide a representative sample or a complete historical narrative over the period. Many of the 23 chosen are not well-known names, although they make a worthy contribution in their individual fields, across education, the military, research, computing etc.




The most famous is Abraham Lincoln, portrayed in his early years solving arithmetic problems in a handmade book, before going on as a young man to profit from the westward movement of the American population in the 19th century. The movement was facilitated by practical skill, which in his case – 'Surveying, and then the law, would raise Lincoln out of poverty, with lawyering for the railroads proving especially lucrative' (p. 31).

In earlier chapters there is a discernible theme of advance from the practical aspect, to mathematics as an established academic discipline. Nathaniel Bowditch is given as an example, having 'mastered the practical details of ocean navigation and coastal surveying' (p. 7), while he 'also acquired a comprehensive understanding of the most advanced European theories on the movements of the stars and planets' (p. 7).

Roberts effectively discusses the subsequent establishment of research departments that were developed in American universities, following the model initially 'pioneered in France during the Napoleonic era and then elaborated by German universities in subsequent decades' (p. 53). He contrasts this model with the already established West Point Military Academy, and its impressive reputation for mathematics tuition.

Roberts addresses the role of conflict as a catalyst for change, the Civil War figuring prominently. He provides 'sketches of the political, military, economic, and social context' (p. 3) that help to give a perspective to the wider environment in which the individuals operated. Kelly Miller is a case in point, as the first African American graduate student. He was at one time the only black professor in the country and an early member of the established civil rights movement.

The advance of technology is a recurring theme in the book, resulting from innovation and development in a number of areas. Roberts discusses Herman Hollerith, who in the late 19th century derived a punched card data storage system which was used in the census to record the population and clarify the position of the expanding westward frontier line. Hollerith's company eventually amalgamated into IBM. Roberts also devotes a chapter to Grace Hopper, who joined the US Naval Reserve in 1943 and went on to become a main player in computer software development.

Frank B. Allen features as one of the more wide-ranging educationalists discussed in the book, a proponent of the New Math school curriculum, an American initiative to address the perception that the country had fallen behind the Soviet Union technologically, following the successful launch of the Sputnik satellite.

The last chapter profiles John Nash, who won the Nobel Prize in economics in 1994 and was made known to the wider public through the film *A Beautiful Mind*. Roberts considers the work of E.T. Bell, a populariser of mathematics whose books were influential to both the young Nash, and to Andrew Wiles who subsequently solved Fermat's last theorem. An informative and worthwhile read.

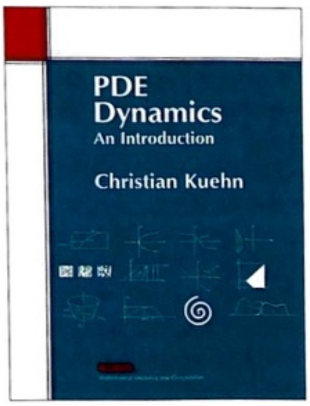
**Wallace A Ferguson CMath CSci MIMA**
Chatham and Clarendon Grammar School, Ramsgate

## PDE Dynamics

### Christian Kuehn

SIAM 2019, 267 PAGES
PRICE (PAPERBACK) £68.50 ISBN 978-1-61197-565-9

**This book is organised into** a preface, a course guide and thirty-six dense chapters. The preface indicates various aspects of the book and requirements for it. The course guide suggests coverage of the book in various possible courses. Each chapter, once expanded in terms of theorem proofs and examples could be the basis of a number of lectures.




The first chapter is a rapid introduction to the subject, it describes various simplifications applied in the book and lists a number of well-known Nonlinear Partial Differential Equations that the book's material could be applied to. The second rapidly covers the qualitative analysis of Nonlinear Ordinary Differential Equations as this subject matter is relatively simpler than the topic of the main part of the book and can form a basis for extensions required for the more challenging material. The third chapter introduces the functional analysis required for the study of Linear Partial Differential Equations.

The fourth chapter commences the true material of the book. Next the Crandall-Rabinowitz theorem is introduced to enable local bifurcations for certain classes of PDE. After this the stability of bifurcations is considered. This leads into spectral theory. Consideration then moves to which classes of non-stationary solutions can be understood, the existence of travelling waves. The waves are now analysed more closely, considering pushed and pulled fronts. The next chapter deals with stability of waves via Sturm-Liouville's ideas. The tenth chapter starts to move towards generalising those ideas.

The text then moves to first order PDE and introduces the method of characteristics and shock formation. Patterns and multiple scales are considered next, with the derivation of amplitude equations. This is followed by proofs of the formal approximations. The proofs led to the advantages of introducing semigroup theory and sectorial operators. Taking this further leads to consideration of dissipation and absorbing sets and the extension of semigroups to semiflows. The next step is to analyse the shape of phase space, in an analogy with similar treatment of ODE. This is followed by the introduction of inertial manifolds and the spectral gap.

Attractors are the next extension considered, followed by the variational equation and exterior product. The bound on the variational equation is used to introduce fractal dimensions. This section has considered the impact of the time variable in the limit as $t$ tends to infinity. This is needed to study attractors. Interesting phenomena are also seen for time away from infinity. This is studied via metastability of the dependent variable for different times. These areas of metastability are referred to as layers which are described as being contained within a manifold. The concept of exponentially small terms is introduced. In the next chapter the concept of coarsening is introduced. This is the idea that the length scale relating to the layers changes with time.

Chapter 23 starts by introducing gradient flows, the first integral of the spatial variable of an evolution PDE. Various kinds of gradient flows are introduced. Convex Lyapunov functionals are referred to as entropy functionals. Some calculations lead to information on the decay of the entropy. These ideas are then combined with those of parameter variation and bifurcation theory, leading to convexity and minimisers. This meant that the minimisers of a functional can be related to a stationary solution of a PDE, with the help of convexity. Removing convexity and introducing the Mountain Pass Theorem leads to non-trivial time periodic solutions.

Chapters 27–32 deal with special classes of PDE and classical effects. The remaining chapters (33–36) are concerned with multi-scale PDE, asymptotic analysis and perturbation.

This book is a very detailed introduction to the topic and so covers many areas with many references to encourage further reading. It is expected that different readers will concentrate on different parts of the book, depending on their interest. It is a very interesting book of its type but readers will need a strong background in Linear PDE as well as some in nonlinear ODE.

**John Bartlett CMath MIMA**

## Enigmaths 192: Solution

| | | | | |
|---|---|---|---|---|
| [1] 4 | 2 | [2] 1 | [3] 4 | [4] 4 |
| [5] 1 | [6] 9 | 2 | 0 | 0 |
| [7] 2 | 7 | [8] 1 | 0 | [9] 5 |
| [10] 6 | 2 | 5 | [11] 2 | 4 |

I = 1, 4; M = 2, 6; A = 3, 5

## Representations of Finite Groups of Lie Type (Second Edition)

### Francois Digne and Jean Michel

CAMBRIDGE UNIVERSITY PRESS 2020, 264 PAGES
PRICE (PAPERBACK) £37.99 ISBN 978-1-108-72262-9


**In pure mathematics,** a group is an abstraction of a set of symmetries of a mathematical object. Despite the simplicity of the formal definition, the study and classification of groups is a vast subject and the various sub-branches of group theory have been the subject of intensive study for most of the twentieth century and beyond (geometric group theory, for example, where one studies infinite groups). The goal of representation theory is to use linear algebra to understand symmetry by representing group elements as linear transformations (the objects and methods of linear algebra being very well-understood and studied).

More advanced is the concept of a group of Lie type. A suitable first definition for such a group is that it is the group of points over a finite field $\mathbb{F}_q$ of a reductive algebraic group over an algebraic closure $\overline{\mathbb{F}}_q$. Groups of Lie type are not to be confused with the concept of a Lie group which is familiar to many mathematicians and theoretical physicists, but there is a close relation as the name suggests: one can view a compact Lie group as the rational points of a reductive linear algebraic group over $\mathbb{R}$. Recall that a Lie group is, roughly speaking, a group equipped with a smooth manifold structure such that the group operations are smooth maps. These groups are extremely interesting and turn up everywhere (in the Standard Model of particle physics, for example).

Chapter 1 begins by setting out some basic results on algebraic groups which are needed for the rest of the text (if you are familiar with some basic algebraic geometry, an algebraic group is simply an algebraic variety equipped with a group structure such that the multiplication and inverse maps are algebraic). Many of the more familiar groups are algebraic (the general linear group $GL_n(\mathbb{R})$, for example).

Chapter 2 outlines the main structure theorems for reductive groups, beginning with definition and properties of Coexter groups and root systems which the student may have learned on other courses. Chapter 3 introduces $(B, N)$-pairs and parabolic subgroups (having two subgroups $B$ and $N$ which form a $(B, N)$-pair being quite a restrictive condition). Chapter 4 outlines Frobenius endomorphisms, Galois cohomology, and the Lang–Steinberg theorem for connected algebraic groups over $\overline{\mathbb{F}}_p$.

Chapter 5 brings in Harish-Chandra theory, Harish-Chandra induction and restriction and the concept of a cuspidal representation. (In general, the work of Harish-Chandra is fundamental for modern representation theory). Chapter 6 introduces endomorphism algebras and Iwahori–Hecke algebras. In the trivial case, the Iwahori–Hecke algebra is simply the group algebra.

Chapter 7 is an exposition of the properties of the duality functor and the Steinberg character. Recall that a character is a function which assigns to each group element the trace of the corresponding matrix in the group representation. Chapter 8