Corrigenda to "Multiple Time Scale Dynamics"

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Abstract

This document is going to collect corrigenda to the book [4]. In particular, typographical and similar more minor errors will be marked in blue while (hopefully not many) mathematical errors will be labeled red. Unfortunately, the existence problem for errors is not very pleasant, e.g., suppose each page is correct with 99% probability and we take a rough page count at 800 pages total then $\mathbb{P}(\text{"no errors at all"}) = (0.99)^{800} \approx 3 \cdot 10^{-4}$ or otherwise said: the probability of no errors in the entire book would be approximately 0.03%. Hence, the existence of this document is unfortunately necessary. Please send me any errors or typos you find and I am going to include them here; please make sure you know precisely how a *correct* version should read to avoid false alarms.

p.45-46,Thm. 2.3.12: The statement (g) in the theorem is apparently not correct as stated since the foliation $\mathcal{F}^u(p)$ (and similar $\mathcal{F}^s(p)$) is only $C^{1,\alpha}$, for under the current assumptions; cf. also the (apparently equally imprecise) statement in [5] to the original hypotheses in [1, 2]. This smoothness issue does not occur for fast-slow systems, so the results in later chapters still hold.

p.55,l.13: Replace $\sup_{w \in w}$ by $\sup_{w \in W}$

p.63,l.16: Replace $\mathbb{R}^2$ by $\mathbb{R} \times \mathbb{R}^2$

p.63,l.-8: Replace $\mathbb{R}^2$ by $\mathbb{R}^2 \times \mathbb{R}$

p.93,(5.9): After the first equality, a minus sign is missing so replace "$= (1+\delta(\ldots))$" by "$= -(1+\delta(\ldots))$"

p.93,(5.10): After the equation replace "with different powers" by "with the same powers"

p.94,l.5: Replace "Example 5.7" by "equation (5.7)"

p.95,(5.17): In the line before (5.17) replace "power series" by "power"

p.100,l.-4: Replace "the inequality (5.3.3)" by the "the last inequality"

p.122,l16: Replace "sufficiently large" by "sufficiently small"

p.122,(6.17): Replace in the vector field in the second equation by $\frac{1}{5}(...)$, i.e., the pre-factor 1/5 is missing in front of the second equation.

p.123: In the middle of the page replace "$0 < y^* < c_a(x_{1,-})$" by "$0 < y^* < c_a(x_{1,+})$"

p.125,l-1: Technically speaking we have to intersect in the definition of S_0 with a compact subset, e.g., a big ball such as $\mathfrak{B}$ defined on the next page to make S_0 compact.

p.129,l-9: Replace "$y_i > 1$" by "$i > 1$"

p.132: In the fourth equation it should read: $R_{11} = \mathrm{d}y_1 \wedge (\nabla g_2 \cdot \mathrm{d}z)$ and $R_{12} = (\nabla g_1 \cdot \mathrm{d}z) \wedge \mathrm{d}y_2$

p.133: In the fifth equation it should be "$\eta_2 = E(X,t) + H(Z,X,t)$"

p.134: The last row in (6.31) in the last matrix should read

$$\varepsilon\left[-g_{2a}Z_1 - g_{2b}X_2 + g_{2y_2}X_4 + g_{1a}Z_2 + g_{1b}X_3 + g_{1y_1}X_4\right]$$

p.165: In the commutative diagram in Proposition 7.1.9 and in the sentence below it, replace "$\phi^{-1} \circ T_1$" by "$\phi \circ T_1$".

p.167: In Theorem 7.1.13, or right before the statement, it might be very help to define "partially hyperbolic" as those hyperbolic objects/points having at least one hyperbolic direction.

p.168,l-4: Replace "are directions" by "are two coordinate directions"

p.168,l-3: Replace "μ_i" by "μ_1"

p.179,(R3): Important typo: replace $c_2 \ln \varepsilon$ by $c_2 \varepsilon \ln \varepsilon$

p.214,(8.41): In the middle column replace y_1 and y_2 by $\dot{y}_1$ and $\dot{y}_2$

p.229,(8.67): The first equality should be an inequality $\frac{\partial^2 f}{\partial x^2}(0,0,0) \neq 0$. Furthermore, in the matrix inside the determinant the lower right entry is a double derivative with respect to y, not x.

p.362-363: The implicit assumption "$g(0,0,y,0) > 0$ for all $y \in \mathbb{R}$" (or in a suitable compact set on which the slow flow is considered) should have been stated explicitly. Add this as assumption (A5) on p.362 and then replace (A1)-(A4) by (A1)-(A5) in Theorem 12.2.3 on p.363.

p.364,l.-3: Replace "$\gamma(\tau_a)$ is $\mathcal{O}(1)$" by "$\gamma(\tau_b)$ is $\mathcal{O}(1)$".

p.366,l.2: Replace "solutions remains" by "solution generically remains"; generic breaking of the slow manifold is again required, similar to the discussion in the remark on p.363, to actually get departure at the buffer point.

p.400,Fig. 13.2: Swap the labels y_1 and y_2 in part (a) of the figure.

p.414,(13.20): Although it is clear from this equation that there are two objects V_1 and V_l, it might be better to change the subscripts so that no confusion can arise here and in related passages of Section 13.6.

p.603,(18.44): Replace "$-\tau \mathrm{D}_2 f(\xi,\xi)+$" by "$-\tau \mathrm{D}_2 f(\xi,\xi) f(\xi,\xi)+$".

p.604,l.6: Replace "$\tilde{F}_\tau(\xi,\tau)$" by "$\tilde{F}_\tau(\xi,0)$".

p.757: The reference [Kue10a] in the book has the title ("Characterizing slow exit points") of an earlier arXiv version of the paper. It should be correctly cited as appearing in the reference [3] below.

p.807: The two entries for "Liénard transformation" should be grouped into one entry reading "Liénard transformation, 9, 573".

References

[1] N. Fenichel. Asymptotic stability with rate conditions. *Indiana U. Math. J.*, 23:1109–1137, 1974.

[2] N. Fenichel. Asymptotic stability with rate conditions II. *Indiana U. Math. J.*, 26:81–93, 1977.

[3] C. Kuehn. Connecting fast-slow systems and Conley index theory via transversality. *Electron. J. Differential Equations*, 2010(106):1–20, 2010.

[4] C. Kuehn. *Multiple Time Scale Dynamics*. Springer, 2015.

[5] S. Wiggins. *Normally Hyperbolic Invariant Manifolds in Dynamical Systems*. Springer, 1994.